

Chapter - 20**Assessment – 1**

After 19 chapters the students will have learnt some essentials of arithmetic. It is assumed in this manual that Functional Mathematics should enable a person to:

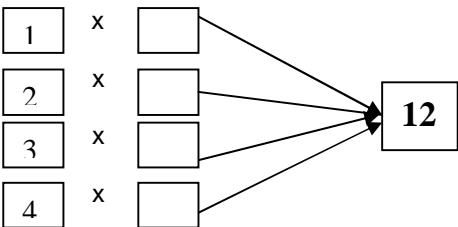
- Do simple operations of multiplication, division, addition and subtraction.
- Do some simple approximations (i.e. sensible guess work)
- Boldly handle fractions with confidence.
- Be at ease with percentages.
- Handle with equal ease both fractions and decimals.

Some ideas on profit and loss and calculations of interest are given.

Now is a good time to assess the student's understanding.

Assorted (and Randomized) problems can be given at this stage.

Random Questions

- List all odd numbers between 1 and 20.
List all even numbers between 1 and 20.
- 83045 – Indicate space value of number 3.
- If $4 + 6 = 10$, then $10 - 4 = ?$
- If $A + B = C$, then $\square + B = C$ $\square =$
If $A + B = C$, then $C - B = \square$ $\square =$
- Fill up:

- Fractions to decimals and percent:
 - $\frac{1}{2} = 0\ldots\ldots = __\%$
 - $\frac{3}{4} = 0\ldots\ldots = __\%$
 - $\frac{3}{10} = 0\ldots\ldots = __\%$
 - $\frac{33}{50} = 0\ldots\ldots = __\%$
 - $\frac{56}{70} = 0\ldots\ldots = __\%$
- Fill up: a. $\frac{2}{3} = \frac{4}{?} = \frac{10}{?}$ b. $\frac{4}{3} = \frac{8}{?} = \frac{20}{?}$ c. $\frac{?}{5} = \frac{9}{15} = \frac{?}{50}$
- $12345 - 2340 = ?$
 - $10005 + 2340 = ?$
- A water tank can be built by 2 people in 3 days. By 6 people in 6 days how many similar tanks can be made.
- Convert the following:
 - 1 meter = cm
 - 1 gram = kg
 - 1 hour = seconds
 - 1 liter = gallons
 - 1 feet = Inches

11. a. 1" = cm
 b. 1 minute = 60 seconds True / False
 c. 1 hour = $\frac{1}{24}$ day True / False
 d. 1 mile = 1.6 km
 e. 1 km = 1.6 miles } which is correct d / e
12. a. $8 \div 2$
 b. $3 \div 3$
 c. 10; 20; 30; divided by ____, ____, ____ gives 1, 1, 1
 d. $\frac{0}{5} = ?$
 e. $\frac{9}{3} = 3$ or $\frac{9}{3} = 27$ Which is correct?
13. a. If $12345 \times 10 = 123450$
 Find 12345×9
 b. If $12345 \times 10 = 123450$
 Find 12345×11
14. Calculate the percentage:
 a. 20% of 350
 b. 15% of 750
15. a. Write in ascending order $\frac{1}{3}, \frac{1}{2}, \frac{1}{4}, \frac{2}{3}, \frac{3}{4}$
 b. Write in ascending order $\frac{1}{2}, \frac{1}{22}, \frac{1}{25}, \frac{1}{250}, \frac{1}{260}$
 c. Write in ascending order $\frac{1}{50}, \frac{2}{97}, \frac{3}{160}, \frac{4}{240}, \frac{5}{255}$
16. Convert the mixed fraction into simple fractions:
 a. $2\frac{1}{2}$ b. $1\frac{1}{4}$ c. $1\frac{1}{2}$ d. $1\frac{3}{4}$ e. $4\frac{3}{4}$
17. Divide the following:
 a. $\frac{22}{5}$ b. $\frac{22}{8}$ c. $\frac{10}{3}$ d. $\frac{22}{7}$ e. $\frac{1}{8}$
18. a. $17 \times 12 =$ b. $17 \times (10 + 2) =$
19. Use brackets in suitable places:
 a. $5 \times 2 + 4 = 14$ b. $5 \times 2 + 4 = 30$ c. $5 \times 2 + 4 - 2 = 12$ d. $5 \times 2 + 4 - 2 = 28$
20. Subtraction of fractions:
 a. $\frac{2}{3} - \frac{1}{3}$ b. $\frac{8}{7} - \frac{1}{7}$ c. $\frac{12}{6} - \frac{8}{6}$ d. $\frac{10}{7} - \frac{3}{7}$ e. $\frac{51}{17} - \frac{34}{17}$
21. Write ascending order: a. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$ b. $\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{1}{6}, \frac{3}{4}$ c. $\frac{3}{5}, \frac{7}{12}, \frac{3}{4}, \frac{5}{6}$
22. a. $\frac{3}{4} - \frac{2}{3}$ b. $\frac{3}{4} - \frac{1}{4}$ c. $\frac{1}{2} - \frac{1}{4}$ d. $\frac{7}{12} - \frac{4}{12}$

[Help: Make the denominators equal, 4 is easy]

23. Which is greater: a. $\frac{1}{2}, \frac{2}{3}$ b. $\frac{5}{6}, \frac{2}{3}$ c. $\frac{5}{7}, \frac{4}{5}$

[Help: Make the denominators equal]

24. a. $\frac{1}{2} + \frac{1}{3}$ b. $\frac{1}{6} + \frac{1}{3}$ c. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$
25. Find LCM:
a. 4, 8, 12 b. 3, 10, 6 c. 3, 10, 6, 5
26. Ramu had 100 sarees. The cost of saree sold for Rs. 300 each. But only 80 could be sold. 20 sarees were found damaged. Profit or loss? What can be done with those 20 sarees, if there should be no loss, no profit?
27. Cost price = Rs. 200. Percent profit wanted = 25%. Sale price =?
28. Onion price Rs. 200 per quintal sold by farmers. A buys and sells to wholesaler B. B sells to Mandy (APMC). D buys from APMC and sells. A, B, C, D all sell for 100% profit. What is the cost price for consumer per kg? (1 quintal = 100kg).
29. Find percent:
a. 5 subjects marks: $\frac{4}{10}, \frac{5}{10}, \frac{9}{10}, \frac{3}{10}, \frac{9}{10}$
b. Total marks 330 out of 600 find % marks.
c. I class is 60%. How many marks total put of 625?
30. Bought a gross (12 dozen) of pens for Rs. 1440 sold at Rs. 10 per piece. Profit or Loss? How much percent?
31. a. Bought a pen for 8, sold at 10. Profit / Loss? Percentage?
b. Bought a shirt for 100, sold at 150. Profit / Loss? Percentage?
c. Bought a cycle for 2000, sold at 1200. Profit / Loss? Percentage?
d. Bought a saree for 500, sold at 300. Profit / Loss? Percentage?
e. Bought a scooter for 16000, sold at 12000. Profit / Loss? Percentage?
f. Bought a house for 1 lakh, sold at 2 lakh. Profit / Loss? Percentage?
32. A Cyclist is moving with 10km constant speed. How much distance can be covered in 40 minutes? (1 Hr= 60 minutes)
33. A box of a 3 dozen-apple cost is 600. What is the cost of 3 apple?
34. If a quintal of rice costs Rs. 1000, what is the price of 5Kg rice?
35. A Bucket contains 10 liters juice. 1-liter juice can be given to 5 boys? Bucket juice can be given to how many boys?
36. A saree costs Rs. 2215. You have only Rs. 10,000. How many sarees can you buy?
37. A pen costs Rs. 23.00. You are given Rs. 200 only. How many pens you can purchase.
38. Bought wholesale 100 shirts for Rs. 10000. Some were small size (300). Others (700) were adult size. Sold the children's shirts at Rs. 50 each. Sold the adult size at Rs. 200 each. What is the percent profit?
39. In 22 above 10% of junior size and 10% of adult size items were found damaged and useless. Now what is the percent profit / loss?

40.

17		1	8	15
	5		14	16
4		13		
10				3
	18			9

All add up to 65.

Fill up diagonals rows columns added = 65

41.

-	1	2	3	4	5	6	7	8	9
1	0	1	2	3	4	5	6	7	8
2	-1	0	1						
3	-2	-1	0						
4				0					
5					0				
6						0			
7							0		
8								0	
9									0

Activity Based Problem:

A. Show by cutting shapes or rectangles or using graph paper.

a. $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

b. $\frac{1}{2} + \frac{1}{4}$

c. $\frac{1}{3} + \frac{1}{3}$

d. $\frac{1}{2} + \frac{1}{3}$

e. $\frac{1}{4} + \frac{1}{8}$

f. $\frac{1}{4} + \frac{1}{3}$

g. $\frac{1}{4} + \frac{1}{3} + \frac{1}{8} + \frac{1}{6}$

Activity Based Problem:B. a. Which is bigger? $\frac{1}{2}, \frac{1}{3}$ b. Which is bigger? $\frac{2}{2}, \frac{3}{4}$ c. Which is bigger? $\frac{5}{6}, \frac{7}{8}$

Show a, b, c above by 2 methods:

1. By cutting rectangles (or Circles).

2. By drawing on graph sheet.

Activity Based Problem:C. Draw any small square. Show $\frac{1}{4}$ in 2 ways (at least).2. Draw any rectangle. Show $\frac{1}{4}, \frac{1}{6}, \frac{1}{9}$ etc3. Draw any circle. Show $\frac{1}{4}, \frac{1}{8}, \frac{1}{6}, \frac{1}{3}$ **Chapter - 21****Substitution**21. Substitution:
This is a very important operation in maths.21.1 E.g.: Start with language: mother tongue, English any other.
What is his name? His name is X

Instead of X put the right name there.
 What is her name? Her name is Y
 Instead of Y put the right name there.

Exercise: Do the following:

My friend's name is

My another friend's name is

Students do the above and compare.

21.2 Activity

- A. Class Activity: Needed – A dictionary
 One student find the word 'substitute' reads out the meaning (local language also ok).
 Another writes on board. Class discusses this. Students can try to give some examples or stories.
- B. Self Study Student's Activity:
 [Students who read this book can and must do all the activities and exercises. Only some activities like games require a class (or teacher or many persons). Many activities can be done by the student]. Take a dictionary. Write down the meaning. Write a few occasions you know in which substitution has taken place.
- C. After doing the activity given above, check whether the following items have been discussed:
 - a. Substitute player in games (Cricket, football etc).
 - b. Substitute teacher.
 - c. Substitute or temporary work.

21.3 Activity

- A. Let the students bring any kind of forms available. Eg their own application form for the course or any other. Let them check one another's forms for correctness.
- B. Teacher can try this:

My date of birth is
 (Sample: 02.10.1869)
 Let the students do this.

Now change the format
 (Sample: 2nd October, 1869)
 Let the students do as per this example.

21.4 Activity

- A. Ganga is a good girl. Let all girl students substitute their own name, in place of Ganga. Let them read out.
- B. Let boys have: Basava is a big boy (or anything else) Do substitution.

- 21.5** If 10 mangoes cost Rs. 60. 1 mango will cost Rs. 6. Now instead of mangoes, put any item. The answer is the same (10 oranges, apples, pens, pencils....). Even unbelievable 10 cars, 10 scooters.
 Students should do ALL these without getting bored (or thinking these are very easy).

10	<input type="text" value="Mangoes"/>		<input type="text" value="Rs. 60"/>
1	<input type="text" value="Mango"/>	Costs	<input type="text" value="Rs. 6"/>

Now put in place of mango [here we ignore singular plural of the nouns. Both are the same for our purpose]. Activity to Do:

- a. b. c. d.
 e. f.

What do we observe here? The box may contain mango, apple or pencil or any other thing, the arithmetic remains the same. Now we understand substitution.

Exercises

- 21.5.1 In a "China Bazaar" (Mysore Local Slang). All items displayed have the same price. You see 1 plastic dabba, 2 pens in a bunch, 4 pencils in a box Take anything for Rs. 10.

- What is the cost of a plastic dabba?
- How much have you paid for 1 pen?
- What is your cost of one pencil?
- In the above china bazaar exercise you did not like the dabba you bought. You go and ask, "Can I exchange this for something". What was the shopkeeper's reply?
- Do you see "substitution", "exchange" etc., have some condition? If yes, what is the condition?

- 21.5.2 Let us go back to (10.5) Mango example. Can we say m = mango, a = apple, b = banana, p = pen, c = car, s = scooter.
 Cost of $10m = 60$. \therefore cost of $1m = 6$.



Exercise:

Students don't be lazy. Write down all the others here.

- 21.5.3 In the above exercise, did you write $10m$, $10a$, $10c$ etc...? If yes, Well Done! Mathematicians also do the same.

Just like them try now with letters of the English alphabet.

$$\begin{array}{ll} \text{If } 10a = 60; & 1a = 6 \text{ (for } 1a \text{ we can write simply } a) \\ 10x = 60; & x = 6 \\ 10p = 60; & p = 6 \end{array}$$

- 21.6.1 We said: 10 of cost  = 60
 1 of cost  = $\frac{60}{10} = 6$

Now let us try:

10 of mangoes cost Rs.60

1 of mangoes cost $\frac{60}{10} = 6$

Exercise:

Students, please write down patiently: In place of write,

Baskets
Dozens
Heaps
bags

Do you see answer is the same? If yes, Well Done!

- 21.6.2 You can have double substitution also, with the same result!
Try

items	mango
Kg	apple
basket	banana
bag	pen

Any combination of two.

- 21.7 We learnt that substitution works when quantities are equal. To write this, symbols can be used. We used and . Some people use \star , \bullet , \ast , Δ etc.,

Note for DTP person. Any symbol is fine. Be careful to write in proper places same symbols.

21.7.1 A.
$$\begin{array}{r} 5 \star \bullet \ast \Delta \\ + \Delta \ast \bullet \star 5 \\ \hline 6 \ 6 \ 6 \ 6 \ 6 \\ \hline \end{array}$$
 Find $\star=?$ $\bullet=?$ $\ast=?$ $\Delta=?$

B.
$$\begin{array}{r} 5 \ ! \ ? \ \% \ \bullet \\ - \bullet \% ? ! 5 \\ \hline 4 \ 1 \ 9 \ 7 \ 6 \\ \hline \end{array}$$
 Find $! , ? , \% , \bullet$

- 21.7.1 For using as symbols, letters of the language are very useful. Why not? What are letters (not a sheet of paper written to another); letters of the alphabet? They are Symbols telling us what sound to produce. "Letters are Phonetic Symbols". Students, ask the meaning of this sentence from teachers (& elders).

- 21.8 Using letters (of the alphabet) as symbols, we can express some mathematical property. If we know, how to substitute, we can do many problems.

21.8.1 We know: $1 + 1 = 2$ 2×1 also $= 2$
 $7 + 7 = 14$ 2×7 also $= 14$
 $(123) + (123) = 246$ $2 \times (123)$ also $= 246$

If we write $\square + \square = \bigcirc$
 $2 \times \square$ also $= \bigcirc$
 i.e., $\square + \square = 2 \times \square$

This is the principle of "multiplication as addition" seen earlier.

21.8.2 Similarly
 $\square + \square + \square = 3 \times \square$
 $\square + \square + \square + \square + \square = 5 \times \square$

21.8.3 Let us use x instead of \square
 $x + x = 2 \times x = 2x$
 $x + x + x = 3 \times x = 3x$
 $x + x + x + x + x = 5 \times x = 5x$
 [x symbol is not used; But $5x$ means $5 \times x$]

21.9 Exercise

21.9.1 Students do this:

- a. Take any one equation: $x + x + x + x = 4 \times x$
 Put $x = 4$
 $x = 6$
 $x = 128$
 $x =$ (any number) Verify.

b. Now write the equation as:

$$x + x + x + x = 4 \times x$$

$$a + a + a + a = 4 \times a$$

$$d + d + d + d = 4 \times d$$

Put $x = 4$ or $a = 4$ or $d = 4$. Verify & satisfy yourself; answers are the same.

21.9.2 $x + 2x = 3x$

Try $x = 1$ $1 + 2 \times 1 = 3 \times 1$ i.e. $1 + 2 = 3$

Try $x = 2$ $2 + 2 \times 2 = 3 \times 2$ i.e. $2 + 4 = 6$

Try $x = 33$ $33 + 2 \times 33 = 3 \times 33$ i.e. $33 + 66 = 99$

This shows that symbols help to generalize.

a. **Exercise:** Do it yourself

$$3y + 2y = 5y$$

b. $5y + 3y = 8y$

c. $5y - 3y = 2y$

[Verify by substituting a number for y . This number can be any number]

21.10 Let us try some substitutions.

21.10.1 If a. $x = 2$ and $y = 5$ then $x + y = ?$

b. $x = 2$ and $y = 5$ then $y - x = ?$

c. $x = 2$ and $y = 5$ then $x - y = ?$

d. $x = 2$ and $y = 5$ then $xy + x^2y = ?$

Ans: a. $x + y = 2 + 5 = 7$

b. $y - x = 5 - 2 = 3$

c. $x - y = 2 - 5 = -3$

d. $xy + x^2y = x \times y + x \times x \times y$
 $= 2 \times 5 + 2 \times 2 \times 5$
 $= 2 \times 5 + 2 \times 2 \times 5$
 $= 10 + 20$
 $= 30$

21.10.2 Exercises:

A. Students can create many such simple examples. Try

B. Given $x = 100$, $y = 99$

Find $x + y = ?$

$x - y = ?$

C. Given $x = 10$, $y = 9$

Find 1. $xy + x =$

2. $xy - x =$

3. $xy - y =$

4. $xy + y =$

D. For fun, do also 1. $x(y+1) = ?$

2. $x(y-1) = ?$

21.11 Activity

21.11.1 Students can do and check with the teacher. Go to addition and subtraction of earlier sessions and do with symbols of earlier sessions.

- 21.11.2 Squares:
A number multiplied by itself is called the square of the number.

Examples:

$$\text{If } x = 2 \quad x^2 = ?$$

$$x = 5 \quad x^2 = ?$$

- 21.11.3 Square, square root, cube, cube root etc can be defined using symbols. This is called algebra. It helps to understand number system also. Any systematic study of science and engineering requires a basic understanding of algebra.
Where there is a formula, there will be an equation.
Where there is an equation there will be algebra.
If there is algebra or equation or formula (or any kind of symbolic representation).
Knowledge of Substitution is necessary

- 21.11.4 **Exercise:** Explain with examples at least 3 situations where substitution is necessary and used.

Chapter - 22

Concept of Negative Numbers

22. Concept of Negative Numbers:

[Teachers! Tell the students that positive numbers and integers can be physically shown, like fingers and objects. Zero is difficult to show. Negative is in the mind, but quite useful].

For self study students: This chapter helps you to understand. It is like reading a storybook. There are no exercise or exams, so please try to read. Some students never read any book, not even storybooks. Such persons neither read English nor their own mother tongue. Such persons will not even come to this para. Teachers (or other elders) may kindly read out and explain this chapter.

For Teachers:

Only very simple concepts are explained. This manual write has tried to make it as simple as possible (for him). Teachers may kindly read and explain to the students.

At the end of the chapter, some games and activities are suggested. Please do them & you can improve upon them too.

- 22.1 Concept of Zero.

- 22.1.1 Zero is a result of taking away what is there.

$$10 - 10 = 0$$

$$\therefore 0 + 2 = 2$$

$$5 - 5 = 0$$

$$0 + 10 = 10$$

$$12345 - 12345 = 0$$

$$0 + 12345 = 12345$$

This tells you that you can take away from a source, a little at a time or all together, until it gets empty. Now, can you take away, MORE THAN what is there? Usually not.

- 22.1.2 If your uncle is a business man and has a current account in a bank, he would have done what is not possible for you and me. i.e., taking out more than what is there.

Of course, he can put in later and bring the balance from negative to zero and may be positive.

[Bank officers will say that your uncle had a debit when he made an overdraft (drew more than the balance)].

- 22.1.3 Now we can understand:

Adding (+ operation) any number to any number is possible. But subtraction is not so.

E.g.: $10 - 4 = 6$

$$4 - 6 = ? \quad \text{It is not possible}$$

Therefore the subtraction grid (of Chapter 3) had some blank pages.

But one can view it this way

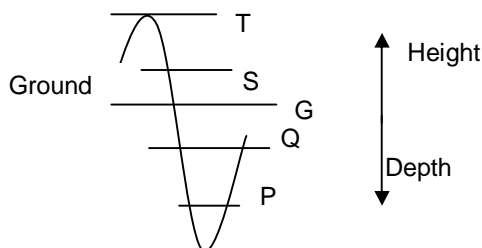
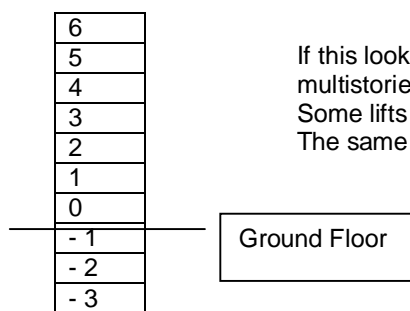
$$6 - 4 = 2. \quad 4 - 6 = -2.$$

- 22.2 What follows is not maths. It is simple explanation. Students, please read through, you will understand algebra better. Teachers may kindly read and explain. There are no problems, exercises or exam, questions on this paragraph. Still explain the concepts to students.

22.2.1 Concepts of depth & height

P – down 4 feet
Q – down 2 feet
G – ground
S – up 2 feet
T – up 4 feet

Down is –
Up is +



If this looks like maths, ask the students to imagine that they are in a multistoried building & in a lift. Ground level is zero (= 0). Some lifts will show B1, B2, B3 i.e. basements. The same can be shown as -1, -2, -3, etc.

- 22.2.2 Concept of Number Line:
Many books give the concept of numbers as points on a continuous line. Where you start (or say you are standing) is zero. All to your right is positive and all to your left is negative. Such a concept is given as a game at the end of this chapter.
- 22.2.3 Concept of a long road:
Some others compare the system of natural numbers to a very long road, where milestones represent integers. Where you are is zero. What you left behind is negative, what is in front of you is positive.
- A. Thus if you go back 2 km and come forward 2 km, you are at zero again. i.e., $(-2) + (+2) = 0$
- B. If you go 2 km reverse, and go still more 3 km reverse, you will be 5 km back from the start. i.e., $(-2) + (-3) = -5$
- C. If you go 1 km forward, and again 4 km further, you will be where?
i.e., $(+1) + (+4) = +5$.
- 22.2.4 The same concepts [i.e., A, B, C given above] can be explained by the “number line” also. Here left is –ve, right is +ve.
- 22.2.5 Concepts are described in this section. They are only analogies [i.e., examples, comparisons] given for the sake of understanding. Working rules come in the next section. They are easy.
- 22.3 Once we know +, - number system we can avoid writing (add) (sub) etc. Adding is just many positive numbers together. Subtracting is coming together of one positive number and another negative number.

To clarify this:

a. $1 + 2 = 3$ Actually $+1 + 2 = +3 = 3$

b. $2 - 1 = 1$ Actually $+2 - 1$ (together i.e. add +2 & -1) $= +1 = 1$

$1 - 2 = ?$ Actually $+1 - 2 = +1 - 1 = -1$ $? = -1$

22.3.1 Example: Write down the question differently including signs of the numbers and then simplify:

a. $18 + 12 = (+18) + (+12) = +30 = 30$

b. $1 + 2 + 3 + 4 = (+1) + (+2) + (+3) + (+4) = +10 = 10$

Exercises:

1. $101 + 99 = ?$
2. Add together 5, 41, 4, 10.
3. What is the sum of Rs. 100, 500, 1000?
4. You are a conductor. You issued 50 tickets of Rs. 5 and 20 tickets of Rs. 3. How much money is in your bag?

22.3.2 Example:

a. $18 - 12 = (+18) + (-12) = +$

+ 18 items
- 12 items

+1
- 1

 $= 0$

$$+ \\ = + 6 = 6$$

b. $1 - 2 - 3 + 4 = (+1) + (-2) + (-3) + (+4) = +(+1-2-3+4) = +(-2-3+4) = +(-5+4) = -1$

Exercises:

1. $101 - 99 = ?$
2. Add together 5, - 41, - 4, 10
3. What is the sum (=final balance) of bank deposits of Rs. 100, 500, 1000 on 3 days and a withdrawal (i.e., taking out money) Rs. 800 later? (Deposit = putting money into your bank account)
4. a. You are a conductor. After getting 10 tickets for Rs. 6 each a passenger gives you a hundred rupee note? How much will you return?
 b. You already had (before issuing the tickets) Rs. 240 in your bag. Including this hundred rupee note, how much will you have?
 c. You had already written on tickets (because you did not have change) change to be given latter. 4 persons have such tickets for Rs. 4, Rs.44, Rs. 94 and Rs. 14 with them. After giving them all how much money will you have?

[a, b, c above look like long questions. But they are really simple. Do and see. You can even ask more questions about the conductor].

22.4 When Positives are together, net total is positive.
When Negatives are together, total is negative

a. $1 + 2 + 3 = 6$ $+1 + 2 + 3 = +6 = 6$

b. $-1 - 2 - 3 = -6$ $-1 - 2 - 3 = -6$ (writing - is a must)

i.e. Numbers carry with them (written before each number) their signs. Fortunately there are only two; positive (+), negative (-). Many times + is not written, it is assumed.

Exercise: Which one is a wrong answer?

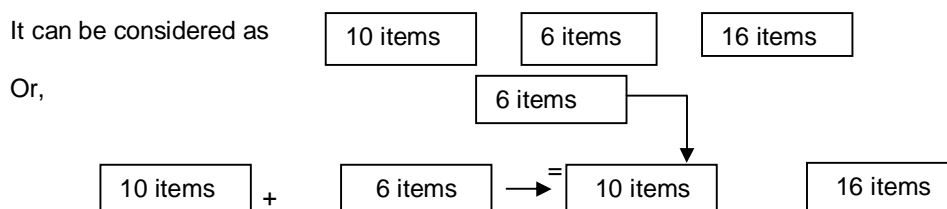
- | | | | | |
|-----------------|----------|-----------|------------|--------|
| 1. $9 + 6 = ?$ | [a. 15 | b. +15 | c. +9+6 | d. 3] |
| 2. $9 - 6 = ?$ | [a. +9-6 | b. -6+9 | c. +3 | d. -3] |
| 3. $-6 - 9 = ?$ | [a. -15 | b. - (15) | c. - (+15) | d. -3] |

- 22.5 (+) and (-) symbols as operations. Explanation of addition process is given below. "Operation" word used here means 'method of doing some work'. Example: let us square 10 means 'squaring is an operation'. This means: Take a number. Multiply it by itself. The result is a square. Thus if we write (square 10) = 10^2 (small 2 sitting on top is a symbol for the operation squaring).

- 22.5.1 Repeat A from 22.5.

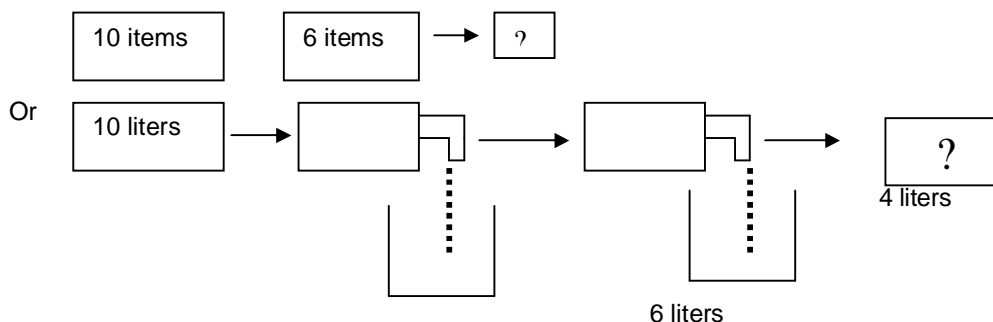
(+) and (-) symbols are also used for mathematical operations (i.e. working with numbers etc). When these symbols and the positive or negative signs come together, then we have to be careful.

If you put 10 items and 6 items together, total is 16 items. This can be written as 10, 6 = 16. But it is not done so. It is written as $10 + 6 = 16$.



This means, in a basket containing 6 items. A more of the same are put in. now the basket has 16 items (all similar)

- 22.5.2 Now consider 6 items taken away from 10 items.



This means, from a stock of 10 items 6 are taken away. Remaining = 4 items.

- 22.5.3 We have used numbers. They are all the same. So no problem. When we come to algebra, we will have different types (or kinds, categories) of quantities.

- 22.6 In the earlier section, we explained the operation of subtraction (-) as **Removing (=Taking Away)** of positive number. This works only when the first number is greater than the second.

Exercise: (Students can do this). Give to primary level children (even 7th standard OK) & get the answers.

- | | |
|-----------------|----------------------|
| a. $3 - 2 = ?$ | f. $100 - 98 = ?$ |
| b. $3 - 3 = ?$ | g. $99 - 99 = ?$ |
| c. $2 - 3 = ?$ | h. $99 - 100 = ?$ |
| d. $10 - 6 = ?$ | i. $1006 - 1000 = ?$ |
| e. $6 - 10 = ?$ | j. $1000 - 1006 = ?$ |

- 22.7 We saw earlier (Number – Line etc...) that if +ve numbers exist (on the right side) –ve numbers also exist (on the left side). If they exist, some of the problems in the earlier section

could be explained. Give -ve numbers also a status. Call them by their status. Thus +6 or -6 you may say $6 = +6$. But these can be a -6. So, let us give them a place.

Exercise:

- a. Point out which of the problems of 22.6 will have negative answer, Explain.

22.8 To make + & - operators under standable, let us assume:


+ve numbers as solid objects.

-ve numbers as holes (of the same size)

When a hole exists as a gap: it is = -1 ●

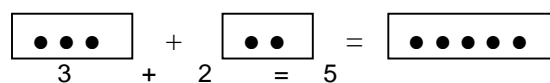
When an items exists as a solid: it is = +1 ∪

If an item sits on a hole: it is = $(+1) + (-1) = 0$

Pictorially ● + ∪ =  = 0

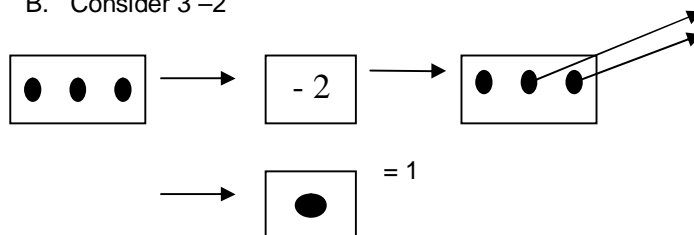
22.8.1 Illustrate (= show by figures)

A. Consider $3 + 2$



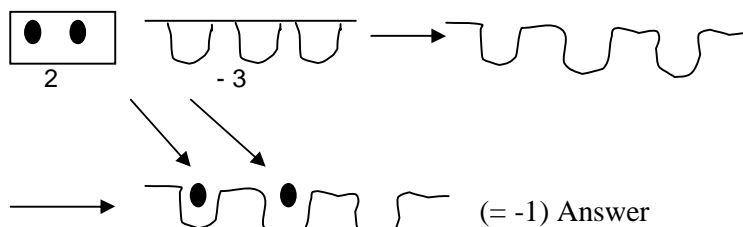
[Teachers! Please do and show. As seen, in the traditional game of 'aLa guLi maNe aata' (ಅಲ ಗುಲಿ ಮನೆ ಆಟ). Using paper or black board does not convey the idea clearly].

B. Consider $3 - 2$



Arrow shows Removal (or Taking Out)

C. Now Consider $2 - 3$

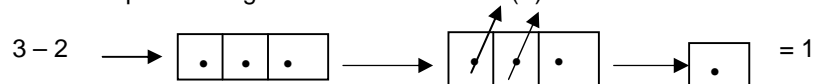


The whole concepts of "holes" was introduced to understand the above viz subtracting a bigger number from a smaller number. Students should go back and see why we wrote $2 - 3$ as $(+2) + (-3)$

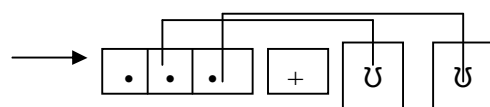
i.e., addition of (positive number) + (negative number)

i.e., ●● + ∪ ∪ ∪

22.8.2 Further explanation: go back and see 22.8.1 (B)



Now see (c) above and write the same as $3 - 2 = (+3) + (-2)$





Exercises: Pictorially do and show:

- a. $5 - 3$ b. $5 - 4$ c. $5 - 1$ d. $3 - 5$ e. $4 - 5$ f. $1 - 5$ g. $5 - 5$
 h. $100 - 100$ [clue: just indicate]

22.9 Standard rules and shortcuts

22.9.1 Adding

$$+ 1 + 2 = 1 + 2 = 3$$

$$+ 2 + 1 = 2 + 1 = 3$$

$$+ 109 + 1 = 109 + 1 = 110$$

$$+ 1 + 109 = 1 + 109 = 110$$

(This can be easily extended to many numbers together $1+2+3+4 = 10$).

22.9.2 Subtraction (only 2 numbers)

$$2 - 1 = 1$$

$$3 - 2 = 1$$

$$4 - 2 = 2$$

this is easy

Now consider

$$2 - 3 = \boxed{+ 2} \quad \boxed{- 3}$$

or $= \boxed{+ +} \quad \boxed{- - -}$

$$= \boxed{\cancel{- -}} \quad \boxed{\cancel{+ -}} \quad \boxed{-} = \boxed{-} = -1$$

$$105 - 5 = 100$$

$$\begin{array}{|c|c|} \hline + + + + +, 100 \text{ items} & - - - - - \\ \hline \end{array}$$

i.e. $\boxed{+ -} \quad \boxed{+ -} \quad \boxed{+ -} \quad \boxed{+ -} \quad \boxed{+ -} \quad 100 \text{ items} = 100 \text{ items}$

$$5 - 105 = -100$$

i.e. $\begin{array}{|c|c|} \hline - - - - -, 100 \text{ items} & + + + + + \\ \hline \end{array}$

$$= \boxed{+ -} \quad \boxed{+ -} \quad \boxed{+ -} \quad \boxed{+ -} \quad \boxed{+ -} \quad \boxed{- \dots - (100 \text{ items})} = -100$$

22.10 Rule: When there are two numbers

- $+, +$ Add the two numbers
 $+, -$ If $+$ is bigger, the result is $+$, after subtraction
 $-, +$ If $-$ is bigger, the result is $-$ after subtracting the smaller number.
 $-, -$ Careful, add both the numbers as if they were both $+$ final result gets a sign $(-)$

22.11 Exercises

22.11.1 Teachers drill the students with many problems.

12.12(a) a. $2 + 5$ b. $2 - 5$ c. $4 - 3$ d. $+ 2 + 5$ e. $+ 2 - 5$
 f. $- 4 - 3$ g. $5 + 2$ h. $- 2 + 5$ i. $- 4 + 3$
 j. $+ 5 + 2$ k. $- 2 - 5$ l. $+ 4 + 3$

12.12 (b) a. $1\ 2\ 3\ 4\ 5 + 5$ b. $5 + 1\ 2\ 3\ 4\ 5$ c. $1\ 2\ 3\ 4\ 5 - 5$
 d. $5 - 1\ 2\ 3\ 4\ 5$ e. $- 1\ 2\ 3\ 4\ 5 + 5$ f. $- 5 + 1\ 2\ 3\ 4\ 5$
 g. $- 1\ 2\ 3\ 4\ 5 + 5$ h. $- 5 - 1\ 2\ 3\ 4\ 5$

(a), (b) above are just a few examples.

Go back to subtraction grid (Para 2.5). With the knowledge of negative numbers, the students could fill up

-	1	2	3	4	5	5	7	8	9
1	0	1	2	3	4	5	6	7	8
2	- 1	0	1						
3	- 2	- 1	0						
4				0					
5					0				
6						0			
7							0		
8								0	
9									0

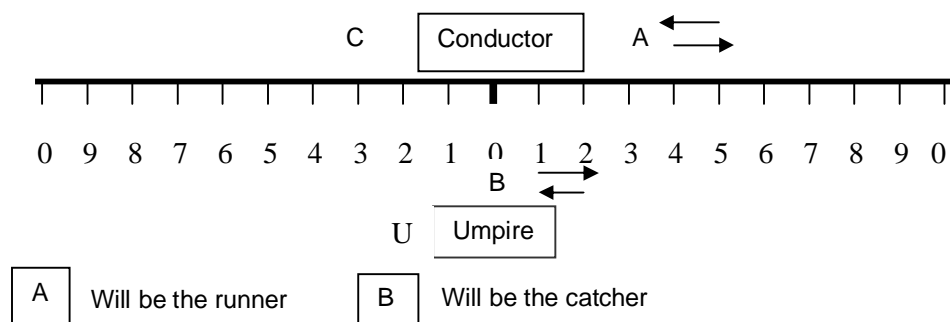
22.12 Here is a game on +ve and -ve numbers. Open filled is better than the classroom.

Game on the number line. This should be played in pairs (2 persons on the field).

One instructor (=conductor)

One umpire (+ Scorer)

Draw a line on the ground (like Kho-Kho)



A will move only on the line and as per instructions of the conductor. B will be blindfolded and stationed at Zero. The distance can be approximately equal to A or B's one step distance. Distance between line A & line B will be less than Arm's length. How to play?

- A – open eyes – B eyes closed (tied) – facing each other. A on line A at zero point. B on line B at zero point.
- C (= conductor will give instructions for moving) Eg: He says 3 to the right; 2 to the right. A goes to #5 on the right. B now moves to catch (A) – like Kho Kho only one direction (i.e., as started) and goes not stopping – until he decides to stop. Then he stretches his hand and says "I got you".
- Umpire verifies and awards a point.
- C will give 3 commands. If B catches 2 points. The game goes up to 5 commands. Total of 10 points.

After the game, class moves to classroom. Then play the same instructions on the board.

$$\begin{aligned} (+3) + (+5) &\text{ i.e., 3 to Right, 5 to Right} \\ &= +8 \text{ (8 to Right)} \end{aligned}$$

$$(+5) + (+3) = +8 \text{ (same)}$$

$$\begin{aligned} (+5) + (-3) &= 5 \text{ to R, then 3 to left} \\ &= 2 \text{ to R} \\ &= +2 \end{aligned}$$

$$\begin{aligned} (+5) + (-5) &= 5 \text{ to R, then 5 to left} \\ &= \text{Starting point} \\ &= 0 \end{aligned}$$

$$\begin{aligned} (+5) + (-8) &= 5 \text{ to R, then 8 to R} \\ &= 3 \text{ left of zero} \\ &= -3 \end{aligned}$$

Can also be

$$\begin{aligned} (-5) + (-3) &= 5 \text{ to left, then 3 to left} \\ &= \text{total 8 to left} \\ &= -8 \end{aligned}$$

$$\begin{aligned} (-5) + (+3) &= 5 \text{ to left, then 3 to R} \\ &= 2 \text{ to left} \\ &= -2 \end{aligned}$$